### Parametric equations assessment

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| 1) | A curve has parametric equations  
\[ x = 2t - 1 \quad y = 4t - 7 + \frac{3}{t}, \quad t \neq 0 \]  
Show that the Cartesian equation of the curve C can be written in the form  
\[ y = \frac{2x^2 + ax + b}{x + 1}, \quad x \neq -1 \]  
Where a and b are integers to be found | [3] |
|---|---|
|  | Rearrange  
\[ t = \frac{x+1}{2} \]  
substitute  
\[ y = 4 \left( \frac{x+1}{2} \right) - 7 + \frac{6}{x+1} \]  
rearrange to  
\[ y = \frac{2(x+1)(x+1)}{x+1} - \frac{7(x+1)}{x+1} + \frac{6}{x+1} = \frac{2x^2+4x+2-7x-7+6}{x+1} \]  
then to  
\[ a = -1, b = 1 \]  | M1 |
|  | Question from 2018 specimen paper |
| 2) | A curve has parametric equations  
\[ x = \frac{1}{t} - 1 \quad y = \frac{2+t}{1+t}, \quad t \neq -1 \]  
Show that the Cartesian equation of the curve C can be written in the form  
\[ y = \frac{a+2x}{b+x} \]  
Where a and b are integers to be found | [3] |
|  | Rearrange  
\[ t = \frac{1}{x+1} \]  
Substitute to  
\[ y = \frac{2+t}{1+t} = \frac{2+\frac{1}{x+1}}{1+\frac{1}{x+1}} \]  
multiply through by x + 1  | M1 |
|  | rearranges to  
\[ y = \frac{2x+2+1}{x+1+1} = \frac{2x+3}{x+2} \]  
\[ a = 3, b = 2 \]  | M1 |
|  | A1 |
| 3) | A curve has parametric equations  
\[ x = e^{2t} \quad y = \frac{2t}{1+t}, \quad t \neq -1 \]  
\begin{itemize}  
  \item[a)] Find the gradient of the curve at the point where \( t = 0 \) \([6]\)  
  \item[b)] Find y in terms of x \([2]\)  
\end{itemize}  
\begin{itemize}  
  \item[a)] First  
\[ \frac{dx}{dt} = 2e^{2t} \]  
by chain rule and  
\[ \frac{dy}{dt} = \frac{2}{(1+t)^2} \]  
by quotient rule  
\[ \frac{dy}{dx} = \frac{\frac{\frac{\frac{2}{(1+t)^2}}{2e^{2t}}}{\frac{1}{e^{2t}(1+t)^2}}}{\frac{1}{1+t}} = 1 \]  
when \( t = 0 \)  
\[ \frac{dy}{dx} = 1 \]  | B1M1A1 |
|  | M1A1 |
|  | B1 |
|  | b)] Rearrange  
\[ x = e^{2t} \]  
to get  
\[ t = \frac{1}{2} \ln x \]  
alternatively  
\[ t = \ln \sqrt{x} \]  
Substitute to  
\[ y = \frac{2t}{1+t} = \frac{\ln x}{1+\frac{1}{2} \ln x} = \frac{2 \ln x}{2+ \ln x} \]  
alternatively  
\[ y = \frac{2 \ln \sqrt{x}}{1+ \ln \sqrt{x}} \]  | M1A1 |
4) A curve C has parametric equations 
\[ x = \sec^2 t + 1, \quad y = 2 \sin t, \quad -\frac{\pi}{4} \leq t \leq \frac{\pi}{4} \]
Show that a cartesian equation of C is 
\[ y = \sqrt{\frac{8-4x}{1-x}} \]
for a suitable domain which should be stated. [4]
### Parametric equations assessment

#### Answers

| Question | Description | Answer
|----------|-------------|------|
| 6) | A large arch is planned for a football stadium. The parametric equations of the arch are $x = 8(t + 10)$, $y = 100 - t^2$, $-10 \leq t \leq 10$ where $x$ and $y$ are distances in metres. | [3]
| a) | Find the cartesian equation of the arch, simplify your answer | [3]
| b) | Find the width of the arch | [2]
| c) | Find the greatest possible height of the arch | [2]
| a) | Rearrange $x = 8(t + 10)$ to get $t = \frac{x - 80}{8}$ Substitute to $y = 100 - t^2 = 100 - \left(\frac{x - 80}{8}\right)^2$ or $y = -\frac{1}{64}x^2 + \frac{5}{2}x$ | M1 M1 A1
| b) | Width of arch greatest when $t = \pm 10$ Substitute to $x = 8(t + 10)$ to get $x = 0, 160$ So the arch is 160 metres wide | A1
| c) | \(\frac{dy}{dx} = -\frac{2t}{8} = 0\) when $t = 0$ Substitute to $y = 100 - t^2 = 100$ so greatest height is 100 metres | [5]

| 7) | The diagram shows the curve with parametric equations $x = t - \sin t$, $y = \cos t$ $0 \leq t \leq 2\pi$ | [3]
| a) | Find the exact coordinates of the points where the curve crosses the x-axis | [3]
| b) | Show that $\frac{dy}{dx} = -\cot \frac{t}{2}$ | [5]
| c) | Find the exact coordinates of the point on the curve where the tangent to the curve is parallel to the x-axis | [2]
| a) | Crosses x axis when $y = \cos t = 0$ $t = \frac{\pi}{2}, \frac{3\pi}{2}$ Points $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{3\pi}{2}, 0\right)$ | M1 A1 A1
| b) | $\frac{dy}{dx} = \frac{y}{x} = -\frac{\sin t}{1+\cos t} = \frac{2\sin \frac{t}{2}\cos \frac{t}{2}}{2\sin \frac{t}{2}} = \frac{\cos \frac{t}{2}}{-\sin \frac{t}{2}} = -\cot \frac{t}{2}$ | M1A1 A1 M1A1
| c) | $-\cot \frac{t}{2} = 0$ or solves to $t = \pi$ point $(\pi, -1)$ | M1A1

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7) The diagram shows the curve with parametric equations $x = t - \sin t$, $y = \cos t$ $0 \leq t \leq 2\pi$
The curve C has parametric equations
\[ x = 2 \cos t \quad y = \sqrt{3} \cos 2t, \quad 0 \leq t \leq \pi \]

a) Find an expression for \( \frac{dy}{dx} \) in terms of t [2]

b) Show that an equation for L is \( 2x - 2\sqrt{3}y - 1 = 0 \) [5]

c) Find the exact coordinates of Q. Showing clearly how you obtained your answers [6]

Substitute \( t = \frac{2\pi}{3} \) to get \( \frac{dy}{dx} = \frac{\sqrt{3} \sin 2t}{\sin t} \) \( \text{ (or simplify to } 2\sqrt{3} \cos t) \)

Substitute \( x = 2 \cos t \) and \( y = \sqrt{3} \cos 2t \) into \( 2x - 2\sqrt{3}y - 1 = 0 \) to get \( 4 \cos t - 6 \cos 2t - 1 = 0 \)

Use identity \( 4 \cos t - 6 (2 \cos^2 t - 1) - 1 = 0 \)

Rearranges to quadratic \( 12 \cos^2 t - 4 \cos t - 5 = 0 \)

Factorises to \( (6 \cos t - 5)(2 \cos t + 1) = 0 \)

Solves to \( \cos t = \frac{5}{6}, -\frac{1}{2} \)

For point Q \( \cos t = \frac{5}{6} \) and \( x = 2 \cos t = \frac{5}{3} \)

and \( y = \sqrt{3} \cos 2t = \sqrt{3}(2 \cos^2 t - 1) = \frac{7}{18} \sqrt{3} \)

**Question from 2018 specimen paper**